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CONVECTIVE HEATING OF A HALF-SPACE (NONSYMMETRIC CASE)

L. N. Germanovich and I. D. Kill' UDC 536.2

An exact solution of the heat-conduction problem for a half-space is obtained in a form convenient for calculations for an important case of a nonsymmetric temperature distribution of a medium bounding a half-space.

The solution of the heat-conduction boundary-value problem

$$\frac{\partial T}{\partial t} = \Delta T\,(r \geqslant 0,\ 0 \leqslant \varphi < 2\pi,\ z > 0,\ t > 0),\ T|_{t=0} = 0,$$

$$\frac{\partial T}{\partial z} = h\left[T - \exp\left(-\frac{r^2}{4\delta}\right)\sum_{n=0}^{\infty}\left(\vartheta_n \cos n\varphi + \tau_n \sin n\varphi\right)\right] \quad (z = 0,\ \tau_0 = 0) \tag{1}$$

written in dimensionless coordinates [1] is sought in the form

$$T = \sum_{n=0}^{\infty}\left[u_n(r,\ z,\ t)\cos n\varphi + v_n(r,\ z,\ t)\sin n\varphi\right] \quad (v_0(r,\ z,\ t) = 0). \tag{2}$$

Substituting (2) into (1) and following [1], we obtain

$$u_n = \vartheta_n \int_0^t I_n(r,\ \tau) f_0(z,\ \tau)\, d\tau, \quad v_n = \tau_n \int_0^t I_n(r,\ \tau) f_0(z,\ \tau)\, d\tau, \tag{3}$$

$$I_n = 2\int_0^{\infty} \lambda e^{-\lambda^2(1+\tau)} J_n(\lambda r)\, d\lambda,$$

where

$$f_0(z,\ \tau) = \frac{h}{\sqrt{\pi\tau}} \exp\left(-\frac{z^2}{4\tau}\right) - h^2 \exp(h^2\tau + hz)\,\mathrm{erfc}\,\frac{z + 2h\tau}{2\sqrt{\tau}}; \tag{4}$$

and $J_n(x)$ is the Bessel function. Using the technique of summation over gamma functions $\Gamma(x)$ [2] and the integral representation of Laguerre polynomials $L_k^n(x)$ [3], we find

$$I_n(r,\ \tau) = \frac{n}{2}(xy)^{\frac{n}{2}} y e^{-x} \sum_{k=0}^{\infty} \frac{\Gamma\left(\frac{n}{2} + k\right)}{(n + k)!} y^k L_k^n(x), \quad x = \frac{r^2 y}{4},$$

$$y = \frac{1}{1 + \tau}. \tag{5}$$

The case n = 0 corresponds to the axisymmetric problem (see [1]). For n = 2p (p = 1, 2, ...) we use the series for the generating function of the Laguerre polynomials [3]. We obtain

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$$I_{2p} = px^p y \sum_{q=0}^{p} \frac{(-1)^q}{q!(p-q)!\,y^q} \int_1^{(1-y)^{-1}} (u-1)^{p+q-1} u^{p-q} e^{-xu}\,du. \quad (6)$$

For n = 2p + 1 (p = 0, 1, ...), we use the relationship between the Laguerre and Hermite polynomials and the integral representation of the latter. We obtain

$$I_{2p+1} = \frac{(2p+1)(2p+1)!}{\sqrt{\pi}(4p+2)!} 2^{2p+2} x^{p+\frac{1}{2}} y^{p+\frac{3}{2}} e^{-x} \int_{-1}^{1} (1-v^2)^{2p+\frac{1}{2}} e^{xv^2} dv \int_0^\infty e^{-u^2} \cos(2uv\sqrt{x}) \left[\frac{d^{2p}}{d\zeta^{2p}} e^{\zeta^2}\right]_{\zeta=u\sqrt{y}} du. \quad (7)$$

Therefore, (3), (4), (6), and (7) permit complete determination of the temperature within the half-space. Questions of convergence are removed if the series in (1) is a finite trigonometric polynomial. Since the integrals with respect to u in (6) and (7) are always taken in elementary functions [4], then the final formulas are sufficiently simple for computations. For instance

$$u_1 = r\theta_1 \int_0^t f_0(z, \tau) \exp\left[-\frac{r^2}{4(1+\tau)}\right] \frac{d\tau}{\sqrt{\tau}(1+\tau)^2} \int_0^1 \sqrt{1-v^2} \exp\left[-\frac{r^2 v^2}{4\tau(1+\tau)}\right] dv, \quad (8)$$

$$\theta_1 = h \int_0^t f_0(z, \tau) \left\{\frac{4\tau}{r^2} \exp\left(-\frac{r^2}{4\tau}\right) + \left(\frac{1}{1+\tau} - \frac{4\tau}{r^2}\right) \exp\left[-\frac{r^2}{4(1+\tau)}\right]\right\} d\tau. \quad (9)$$

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